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ABSTRACT

Overall goals and trends in mathematical education are discussed, with emphasis on the question "why teach mathematics?" In an introductory chapter, the philosophy of schooling in various countries is considered, with the suggestion that reasons for teaching mathematics are political as well as philosophical. The historical role of mathematics in schools is considered more extensively in the second chapter. In Chapter 3, the changing content and changing attitudes toward mathematics are discussed. The next chapter considers the expectations from mathematical education in various societies, and the concluding chapter summarizes the paper in terms of conflicting utilitarian and mathematical objectives. (MS)

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OVERALL GOALS AND OBJECTIVES FOR MATHEMATICAL EDUCATION (*)

by

Ubiratan D'Ambrosio

(*) Invited report presented to the Section B-3 of the 3rd International Congress on Mathematical Education, Karlsruhe, Germany, 16-21 August 1976.

OVERALL GOALS AND OBJECTIVES FOR MATHEMATICAL EDUCATION

(Abstract)

by Ubiratan D'Ambrosio

In the paper we discuss the overall goals and trends in mathematical education with a very significative subtitle: "Why teach mathematics?".

The paper, due to obvious limitations of space and time to prepare it, can not be but an introduction to our approach to the theme. It raises issues and refers to sources where our approach finds its supporting basis, as well as to sources for a more conventional treatment of the subject. We hope the paper generates positive anguish among mathematical educators, stressing our responsibilities and confidence in the power we have, as educators, in shaping the future.

We focus the question of the subtitle from the point of view of development of society, of school systems, of teacher education, as well as of the changing views of the nature and role of mathematics, and on the nature of the teaching-learning process. We refer to the role of mathematics for the individual, keeping in mind his place in society. We stress the relation between general education goals and the goals and objectives for mathematical education, dealing with them within a general context, and placing them in different models of societies and various stages of development.

The paper has five chapters : 1. Introduction ; 2. Mathematics and school ; 3. The role and nature of mathematics ; 4. What does society expect of mathematical education? 5. Again mathematics and school; plus a final part on References and Bibliography, listing about 100 items and identifying sources.

Instead of concentrating on specific objectives and goals for mathematical education, we opted for an analysis of the place of mathematics in the present and in the future, drawing many conclusions by the inter -

pretation of the past, and its necessary implications on mathematical education.

We recognize a total interaction between science, in particular mathematics, and society, not only from the historical perspective, accepting mathematics development as a result of socio-economic factors, but also accepting that mathematics, though less apparently and immediately perceived as in the case of natural sciences, affects in an ever increasing degree the whole pattern of thought, culture and politics. We take into account a rapidly changing educational pattern all over the world, mainly due to the rapid concretization of the ideal of mass education. This is inserted in a context of pursuing the concretization of the ideal society, which though taking different forms and shapes keep an entire agreement on some of the basic issues, which include the tolerance of a minimum disparity between power and privilege. The resulting conflict between educational growth and educational disparity can be summarized into three different philosophies underlying the entire study of objectives and goals for mathematical education : (i) the search for values (ii) the pursuit of new knowledge (iii) the support of a designated social structure. It is a mere palliative to improve the existing educational model without a global analysis of school as an institution and science as a needed activity and an element in improving the quality of life.

The definition of priorities in education and in science poses major problems for developed nations and gigantic problems for developing countries. We accept education, in particular science and mathematical education, hence scientific research structure, as a socio-cultural phenomenon, anchored on cultural values; therefore its aims have to be analised in the global context of national priorities and goals and to depend less on the intrinsic structure of science itself. On the other hand, the political and economic structure of the world, and the growing interdependence of the various countries, nations and societies, make it absolutely necessary for emerging countries to close the so-called technological gap with developed and industrialized countries, and develop a sound and capable intelligentsia. To conciliate the goals, essential for emerging countries as well as for smaller developed countries, of engaging

in a dignified and honest way of relationship with bigger developed countries, and at the same time preserving traditional socio-cultural and moral values, seems to be the major problems facing educators all over the world, both from developed and developing countries. In this context we have to analyse science, in particular mathematics, and education, in particular mathematical education.

We claim mathematicians share the responsibility of bringing better days to fellow human beings. But some see in the very nature of mathematics a danger as an oppressive tool. This danger, present in developed and industrialized societies, is of concern for countries which, having achieved political independence, are now beginning to struggle for economic and cultural independence. This concern must be associated to the inadequacy of a structure of education, designed for an elite and now being applied to the mass of the populations. Probably, in understanding this fact resides the key to the preservation of mathematics as an important subject in school, specifically in reassessing its position as an independent subject in general education and inserting it into the broader concept of culture, of more value than the mere knowledge associated with it.

We see the educational process as the conjugation of global socio - economic aspects aiming at the betterment of the quality of life. In this conjugation intervene the same as in the technological process, the philosophy to which society subscribes, as well as considerations about manpower and available material resources.

We distinguish several forms of education: a purely vital education , instinctive, through which children learn how to survive, and to continue the species; a societal or behavioural (in the lay sense) education, in which children are taught basic attitudes and conduct, and acquire moral values; an artisanal or professional education; and a contemplative or speculative education. We concentrate our analysis on the last two. In fact, we identify instances in which the two forms join efforts and new knowledge is created. Mathematics is a good example of this link between the two patterns of education.

The very beginnings of numbers show a strong practical component as well as reflections on the purely contemplative aspect. Historical considerations lead me to displace counting as a major component of mathematical education in the context of a contemplative or speculative scheme. Although taken for granted as part of theoretical mathematics in western culture, and somewhat misplaced in medieval schools, ordinary arithmetic as the core of mathematical studies has to be reexamined.

The development of a new economic order towards the end of the Middle Ages, with the growing importance of towns, trades and industry, forced a deviation from the economy of feudalism. At the same time, the basis for the Industrial revolution was laid, heavily based on the springing up of the new science, experimental and quantified. Another important factor was the opening of the frontiers of the world through navigation and conquest , introducing new and cheap materials into an economy geared increasingly toward the processing of those materials.

In the period going from the Middle Ages through the nineteenth century, schools went through radical changes. We see the appearance of universities and at the same time the development of what we have called artisanal or professional education, resulting from a form of professional guild in a parallel structure. In what refers to mathematics, its place in education of this period is quite weak.

We thus reach the end of the nineteenth century, and enter the twentieth century with strong motivation for mathematical research, derived from many sources. Much of what dominates mathematics research today can be traced back to last century. More than significant new ideas in mathematics, we see the richness of new fields of applications, anticipating a fast and profound change resulting from the use of computers by mathematicians. In the same vein, we do not see significant changes in schooling. There is a major change, which is mass education, but the approach to education is practically the same as it was in the last century, always focused on "how much children learn", and seeing that "children behave according to a certain pattern". There is a lack of

emphasis on creative work.

Paradoxically, there is a profound richness of new directions which science and society are taking. No doubt we are living through a second scientific revolution, with news fields of research being opened; new tools for the understanding and control of nature, both in smaller and larger dimensions than man has ever thought, are being created. We can not see mathematics and mathematical education unrelated to these new directions, as well as to social and economic changes occuring in the world.

The main objective of mathematical education which we advance throughout the paper is to develop the ability of individuals to identify mathematics in intellectual experience, and distinguish mathematical reasoning and method in all the situations where they are either present or can be potentially inserted. Mathematics is placed within the ample context of scientific reasoning and method, and related to other languages which try to simulate a reality. The process of mathematical creation proceeds like any form of creation, as the result of some internal force which we can not identify while the object of mathematics is a combination of sensorial experiences with some process of abstraction and correlation of ideas. We recognize a trend towards a new relationship between mathematicians and non-mathematicians, and as a result the appearance of new foci of interest for research mathematicians. This new relationship goes both ways and the appearance of completely new scientific fields, bearing mixed characteristics of mathematics as we know it today, and of other disciplines is quite predictable. The role of computers in those new scientific disciplines will probably be a major one.

The implications of this reasoning in defining the objectives for mathematical education are of considerable magnitude. Certainly, the emphasis should be changed from what is taught, that is, from curricula, syllabi and contents, to methodology and to a new classroom environment, both geared towards creativity. The main objective is to give mathematics a new dimension that will fit a new world and societies which are closer to each other, and yet confronting each other. We need new mathematics, and all the creative power

of youth to generate new forms of thought which we can not envisage. The primary objective of mathematical education should not be the perpetuation of knowledge, or to push a little further existing knowledge. This know - ledge will go on or fade away per se . But the primary objective of mathematical education is to foster the creation of new knowledge. At the same time, the need for making existing knowledge available to a growing number of professionals, - and this affects in particular developing countries - should be provided in a parallel structure, basically not differing from what happened in the Middle Ages through the industrial revolution. A major peda - gogical effort should be promoted to bring to a larger number of individuals and to be made available to developing countries, through educational methodo- logy, using audio-visual and computarized technology, advanced mathematical theories. This can not be achieved through pre-structure curricula. A dynamical and flexible approach to contents is needed with considerable emphasis on methodology of access to existing knowledge, in a readily accessible, under - standable and applicable form. This approach responds more immediately and rea- listically to the needs of developing countries, as well as to the changing role of mathematical education in the changing pattern of developed and industrialized societies.

We believe a sound strategy of reform is to implement it in all levels simultaneously. Certainly, changing the attitude of teachers and moving the focus of teaching from what is taught to the learner, is an essential preamble to any improvement of education. A realistic and dynamic approach to change means, in its essence, a built-in mechanism of change, in which teachers and learners are brought into a single organic component.

With a more dynamic and unbiased attitude, we believe mathematics can play an important role in education, and contribute to the fullfilment to the primary, essential and undisputed goal of education which is to enhance the betterment of the quality of life of mankind.

OVERALL GOALS AND OBJECTIVES FOR MATHEMATICAL EDUCATION

by Ubiratan D'Ambrosio

In this paper we will discuss the overall goals and trends in mathematical education with a very significative subtitle: "Why teach mathematics?". We will first explain the paper itself and its structure, as well as the way it was written.

The paper, due to obvious limitations of space and time to prepare it, can not be but an introduction to our approach to the theme. It is meant mainly to raise issues which are not commonly emphasized in stating goals and objectives of mathematical education, and to give references to sources where more traditional approaches can be found. We hope this paper will generate positive anguish among mathematical educators and will generate more confidence in the responsibilities, and above all in the power we have, as educators, in shaping the future.

The paper was initially designed to center in the question of the sub-title from the point of view of development of society, of school systems, of teacher education, as well as of the changing views on the nature and role of mathematics, and on the nature of the teaching-learning process, referring to the role of mathematics between general education goals and the goals and objectives for mathematical education. These topics should be dealt within a general context, including their placement in different models of societies and various stages of development.

The outline of the paper was drafted in September 1975 and first discussed in December 1975. The revised outline called for five chapters. We have kept the aims of each chapter, while moderating their initial broad reach for obvious reasons. In addition to the five chapters, there is a section for References and Bibliography. The chapters and respective summaries, are:

1. Introduction: Discussion of some concepts and general objectives of education. Interpretation of the institution "school" in the social context, with emphasis on the aspirations of newly independent countries, but also examining

some of the crises through which school and science education are passing all over the world. Such considerations will reappear in subsequent chapters.

2. Mathematics and school : Accounts of the development of mathematics and of education serve as introduction. This chapter situates mathematics in the educational context through various important periods of history. The division is rather personal, and corresponds to what we consider turning points in history which influenced the teaching of mathematics, even though these may not correspond to turning points with respect to mathematical discovery. These periods go from the sources to the middle ages, when universities were instituted; from the middle ages to mid XIX-century, when the industrial revolution occurred and also colonies began to play a fundamental role in the socio-economic process of developed countries; from mid XIX-century to the 1950's, when science and technology laid the ground for a profound social change, following the end of World War II, when the entire world equilibrium was changed with the emergence of more than half of the world population as independent countries, with specific problems and goals. Also, some explanatory considerations about the future are pertinent to this chapter.

3. The role and nature of Mathematics : A discussion of the changing contents of mathematics and the changing attitudes towards mathematics. Some epistemologycal considerations, as well as a discussion on the process of mathematical creation are placed in the context of general science, technology and society. The important question of the values of mathematics, for example if we teach mathematics to develop a capability of reasoning, a logical mental structure, or merely to be useful, are dealt with. Also, how does mathematics relate with other fields and the emergence of new fields of application of mathematics are discussed. Some discussion on the direction mathematics is going is part of the chapter. Some different views on this are reported, with enough care not to give any idea of conclusiveness. The fundamental importance of the topic in the definition of global education policies is made clear.

4. What does society expect from Mathematical Education? We discuss scientific and technological aspirations of developed, industrial societies, and developing countries, and try to examine the difficult question of how these

views can be conciliated. The attacks on science, and particularly mathematics, are examined. Views of attackers and defenders, moderate and radical, are discussed, emphasizing contradictory aspects of the various opinions. Finally, to close the chapter, a summing up of the above is presented, illustrating the difficulties of the topic and inviting further discussion on social responsibilities of mathematics.

5. Again mathematics in school : This chapter puts all the ideas discussed in the previous chapters in the general context of education. All levels of education and the building up of research trends are discussed. Preliminary to the role of mathematics in the general context is the question of what kind of future we are building up. Some school systems are discussed and some of the ideals of different kinds of societies are looked upon from the point of view of identifying what role mathematics does play in the educational policies of these societies. Hints of how mathematics should be dealt with in those contexts are brought into consideration.

The revised outline was sent to about 150 individuals all over the world, among them panel members, members of the Executive Committee of the International Mathematical Union, members of ICMI's Executive Committee, well known mathematicians and less known younger mathematicians, both from developing and developed countries, physicists, engineers, and other scientists, as well as humanists and professionals of various fields. It is our feeling that the "man in the street" who once in his life-time went to a school and was taught some mathematics has something to say on what he got out of this, and in which way he regards what his children are being taught in the school. Some 30 individuals reacted, plus some others who, either from receiving the programme of the Congress, or by learning through others of the work under way, sent me me some comments on the subject. The geographical distribution of the reactions was somewhat disappointing showing much less concern among individuals from countries where education is such a vital activity, both from the point of the expectations of the educational system and also from the enormous share of the national economy there involved, than from individuals from countries where it is a matter of just making better a system that undoubtedly has served and still

serves its purposes. This pattern of responses reveals the mimetic attitude that permeates education in developing countries, where reforms and trends are adopted without adequate criticism, without much sense of national priorities and values.

1. Introduction:

Since the first reactions to the outline were received, it was clear that the ambitious reach of the topics covered and the depth we intended to cover them had to be moderated. Undoubtedly, the course we proposed to follow would require much more time than we were able to devote to the project, and much more space devoted to this paper in the proceedings of the Congress, as well as in the volume "New Trends in Mathematical Education IV", to which the revised version of the paper is destined. We moderated the reach of our paper without changing its structure and hoping that some topics and issues of the paper will be rather provocative for further work in the subject than itself a study of the topics and issues. Ideally, this paper should originate some essays on the many points raised and which, we believe, deserve further analysis.

As mentioned above, the structure initially designed was kept. This structure departs from some of the traditional approaches to goals for mathematical education. The goals are casually presented in a way which we would rather classify as common sense goals. For example, the listing given by Stephen Krulik and Ingrid B. Weise ([Krulik Weise]) is quite comprehensive and summarizes what is generally agreed upon. The authors put them as:

"Goal 1: To achieve for each individual mathematical competence appropriate for him

Goal 2: To prepare each individual for adult life, recognizing that some students will require more mathematical instruction than others

Goal 3: To foster an appreciation of the fundamental usefulness of mathematics in our society, particularly with reference to understanding and improving man's environment

Goal 4: To develop proficiency in using mathematical models to solve problems".

We found this listing of goals much more uncompromising than many others and borrowing words from A.N. Whitehead (in [Whitehead Aims], deprived of a

to exhibit pedantry and routine. In fact, statements of goals for mathematical education in general can not depart from overall goals of education. And the above statement of goals fits much of what we get from the philosophy underlying the Faure Report [Faure Report] .

Instead of concentrating on specific objectives and goals for mathematical education, we opted for an analysis of the place of mathematics in the present and in the future, and its obvious implications on mathematical education, thus responding to the subtitle "Why teach mathematics?". The importance given by Professor K. Kuratowski in his account of mathematics in Poland [Kuratowski] to the moods and tendencies of the Polish society as a factor to be taken in the birth of Polish school of mathematics encouraged us to look into this kind of Sociomathematics, which might side with the goals of the new discipline of Sociobiology (see [Sociobiology] or Socio-linguistics. We recognize a total interaction between science, in particular mathematics, and society, not only from the historical point of view, accepting mathematics development as a result of socio-economic factors, but at the same time accepting that mathematics, though less apparently and immediately perceived as in the case of natural sciences, affects in an ever increasing degree, the whole pattern of thought, culture and politics. The lucid introductory chapter on "The Emergence and Character of Science" of J.D. Bernal's Science in History, very clearly illustrates instances of this. [Bernal] .

This essay has the objective of an overall analysis of objectives and goals in the various countries and regions of the world, identifying and possibly interpreting trends. It deals with the fundamental question of "Why tech mathematics?" in view of a rapidly changing educational pattern all over the world, mainly due to the rapid concretization of the ideal of mass education. It is universally accepted that the benefits of education, not only in the elementary grades, but also in professional and scholastic fields, both in formal schooling and in some form of continuing education, should extend to the lower economic and social strata of society. It is generally accepted that literaly everybody has the right of access to education, to the higher possibilities of individual own capacity. It is also a universally accepted ideal that society be constructed in such a way that only a minimum of disparity between power and privilege is accepted.

This conflict between what we would call, following Johan Galtung, educational growth and educational disparity, [Galtung] , meets with serious questioning about the value, and even in a less radical view, the pattern of this general or mass education which humanity is beginning to achieve.

We all know about the growing movement called "Survival" and its various similars and ramifications (see [Auto-Critique]), but much more influencial and of immediate consequence is the pattern of educational reform going on in several countries. We see an ideal of mass education in developing countries well expressed by Ahmad H. Ubeid: "It is our task in education to provide every young person with, at any rate, sufficient instruction to enable him to embark in life in society without too much difficulty. He must be given an education that will help him to determine his needs and to participate in his own instruction".(italic is ours) [Ubeid] . This does not differ essentially from what Mme. Alice Saunier-Seïté means by saying "we cannot have in the beginning of [professional] life a situation which corresponds to the level of the diploma. What is important is to obtain it a few years later" [Saunier-Seïté], or from what is today called the "Chinese experiment", as described for example in [China], or from what is in the core idea of the British experiment, studied in Section A-5 of this Congress, or the Cuban model of approaching students and peasants in what was called an absolutely revolutionary paedagogical method by Jean-Jacques Alphander [Cuba], or the rationale between the joint program established by the Organization of American States and the Brazilian Ministry of Education for the development of leadership in Science and Mathematical Education in Latin America [D'Ambrosio Montevideo].

The association and conflicts there resulting from three different philosophies of education: (i) the search for values (ii) the pursuit of new knowledge (iii) the support of a designated social structure, permeates the entire study of objectives and goals for mathematical education. This search for a balanced approach is crucial in developed and industrialized countries, with somewhat stable social structure, as it is so well exemplified by the "Report of the Carnegie Commission on Higher Education", and the criticism it raised. I bring special attention to the appraisal of it by Donald McDonald [McDonald] and the reaction to it by members of the Commission and others [Carnegie Reaction] as very representative of what we might identify as a need for some break-through

ideas in education. It is a mere paliative to improve the existing educational model, without a global analysis of school as an institution and science as a needed activity and an element in improving the quality of life. What is low quality of life is absolutely and very clearly illustrated by the African farmer who tries to sell his produce in the trading post, so dramatically and irreverently described by Céline in his "Voyage au bout de la nuit". Or a more scholarly definition of quality of life as a valid criterion for political, social, economic and scientific action can be found in the essay of Horst Pommer [Pommer] . Bertold Brecht dramatized the responsibilities and the motivation of the work of a scientist as a means of improving the quality of life as found in the retrospective capsule of the life of Galileo Galilei. In the same vein, President Julius Nyerere, of Tanzania compares the young African who has the opportunity of receiving an education to a young member of a starving village to whom is given all the remaining available food so that he can get stronger and go out to look for nourishment for the survival of the village.

If the problems of defining priorities in education and in science poses such major problems to developed nations, we can easily understand the gigantic dimensions of the problems posed to educators, administrators and governments of developing countries, when allocating badly needed resources to education and scientific development. While there is a major convergence of viewpoints regarding the values and priority of free and nondirected research in developed countries, with great emphasis in university research, as was so well advocated by the President of the German Research Society, Dr. Heinz Maier-Leibnitz [Maier-Leibnitz], its priority for developing countries is more and more often being questioned (see for example [Moravcsik] and [Wade]). Undoubtedly, this is such a major decision affecting governments of emerging countries that no attempt other than the overture to diversified educational experience should be accepted. Indeed, developing countries have a much better possibility of achieving adequacy and excellence of their educational and scientific structures out of the competition of diversified options than by following a unique approach. The well known incongruencies, difficulties and inadequacies of technological transfer are enormously greater when we talk about the transfer of educational models, including

the structure of scientific research. It is a major premise that education, in particular science and mathematical education, hence scientific research structure, is a socio-cultural phenomenon. Its structure, hence teaching modes, curricula, evaluation schemes, all are product of socio-economic and cultural frames, guided by national priorities and goals, and less dependent on the intrinsic structure of science itself, certainly even less dependent on the philosophical and psychological framework of foreign societies. On the other hand, the political and economic structure of the world, and the absolute interdependence of the various countries, nations and societies, with the resulting competitive game, which we accept ideally in the most sane acceptance of the concept of competition, makes it absolutely essential for emerging countries to close the so called technological gap with developed and industrialized countries. In order to be incorporated in the concert of free and independent nations, and to relate economically and culturally with dignity with bigger countries, emerging developing nations need a sound and capable intelligentsia. To conciliate these goals, that is entering in a dignified and honest competition with other countries and at the same time preserving traditional socio-cultural and moral values, seems to be problem facing educators in all countries, both developed and developing, both rich and poor, both small and large. In this context is inserted science, in particular mathematics, and education. Within these overall goals we will look into the theme of this panel.

Clearly, any consideration about "Why teach mathematics?" is deeply political, in the sense of at least an educational policy. I can not agree with a highly respected German university professor who answered me saying that he sees no global planning in defining the objectives of mathematical education. Neither can I accept the view that a set of recommendations put out by national authorities or organizations or advisory groups of certain countries can determine the future of mathematical education throughout the world. On the other hand, I do agree that in fact these recommendations and national educational policies of the various countries will influence the overall picture of mathematical education throughout the world, and will imprint this influence in future generations. No doubt countries with great resources, both intellectual and financial, to generate

independently their own educational structure, will rely much less on what comes out of congresses like this one, while countries that we classify in the spectrum of less developed will rely much more on what is discussed in international conferences to set their national programs. As a consequence, a paper like this should not be conclusive nor authorative in its tone, and should avoid the attitude of placing absolute values for mathematical education. Although quite old, the following quotation illustrates an unfortunate common view of the values of mathematical teaching: "The real reason for learning, as it is for teaching algebra, is, that it is a part of Truth, the knowledge of which is its own reward" [Hudson] . Instead, this paper should try to understand and place in its appropriate socio - cultural setting some of trends we identify in analysing the different objectives and goals of mathematical education. To place mathematical education in the adequate socio-cultural setting seems to be of the most fundamental importance.

As was put by E.W. Hamburger in his trend paper "Physics and Recent Trends in Education" in the International Conference of Physics Education, which met in Edinburg in 1975, [Hamburger] , we agree that education and science education in particular is believed to promote aims such as democracy, development, equality, tolerance, creativity, independent critical thinking, scientific attitude, de - mystification of science, etc. And we also put the question : does mathematical education as practised today further these aims? Can it further them and is it pertinent to them? In fact, these are the basic issues raised in the Faure Report. Strongly contrasting with the well known remark of G.H. Hardy - in view of the misuse of science -that the results of one's research should lead, as far as possible, away from practical applications [Hardy Apology], we claim mathematicians, as well as scientists in general, share the responsibility of bringing better days to fellow human beings. Indeed, some see in the very nature of mathematics a danger as an opressive tool. See for example the book of Else and Jens Høyrup [Høyrup] and the review of it made by John Lamperti [Lamperti] . This danger, present in developed and industrialized societies, and strongly denounced by a growing number of mathematicians (see [Varsavski] and [Porquoi?]) and resulting, in spite of all the defense against such attacks, in a real decline of the position of mathematics in education, is indeed of a great concern for countries which have achieved

political independence and are now beginning to struggle for economic and cultural independence. In fact, science is foreign to the culture of most underdeveloped countries and, as it was clearly put by E.W. Hamburger in the cited paper, one can draw an analogy between the influence of scientists in the underdeveloped countries today and that of the religious missionaires in the past. The aim of erradicating old ways of thinking and to replace them by new ones, brought ready made from developed countries, may act as a very effective tool of a new form of colonialism, as we have discussed in [D'Ambrosio Lettre]. And we draw the same parallel between science and mathematical education, which was designed for an elite some time ago and now is being taught to the mass of citizens. Probably, there resides the key to the preservation of mathematics as an important subject in school. The key resides exactly in reassessing its position as an independent subject in general education and inserting it into the broader concept of culture, of much more value than mere knowledge. If we accept the very broad definition of "education as the acquisition of the art of the utilisation of knowledge", as given by A.N. Whitehead (see[Whitehead Aims]), then we are led to a dynamical and flexible approach to mathematical education, putting more emphasis in a strong motivation for mathematization or "mathematicking", using the neologism introduced by J.H. Mason and J.E. Baker [Mathematicking] through an integrated approach as defended for example by R. Ahmed in his lecture for the plenary session on "New science and old cultures", of the International Conference of Physics Education, Edinburg, August 1975, or in [D'Ambrosio Caracas] followed by the development of strategies of utilization of advanced theories and results. For more comments on this, see [D'Ambrosio Integração] and [D'Ambrosio Rabat].

Summing up, we may define the educational process as the conjugation of global socio-economic aspects aiming at the betterment of the quality of life. In this conjugation intervene, the same as in the technological process, the philosophy to which society subscribes, as well as considerations about manpower and available material resources.

In the subsequent chapters we will look specifically into mathematics and mathematical education, past, present and future.

2. MATHEMATICAL AND SCHOOL.

We distinguish several characteristics of education, which may serve as a guide for a brief historical account of it. We distinguish a purely vital education, instintive, through which children learn how to survive, and to continue the species, which has been traditionally put under the responsibility of families or structures which remind us of a family environment, and a societal or behavioural (in the lay sense) education, in which children are taught basic attitudes and conduct, and acquire the first moral values, also traditionally placed in the hands of family strucutre or the like. Maternal and child care centers we understand as educational structures of a family type. These are the basic components of what Margaret Mead would call a post-figurative model (see [Mead]), and which we find present in practically all educational structures. Then we distinguish two other forms of education, which we distinguish as artisanal or professional, and contemplative or speculative. In the first category we place Greek, mainly Spartan, and Roman models, as well as the Chinese public service structure, all the civilization of antiquity, and all the initiation cultures which prevail to nowadays mainly in Africa. At the same time, we notice an intermingling of the second category, represented firstly by religious practices and motivated by understanding the place of man in the world, and certainly the product of a certain economic and social stability in its first manifestations. The sophists are quite illustrative of this remark, and to a certain extent Roman courtesan life. Certainly, in this intermingling we will identify the genesis of scientific ideas. As it was quite clearly put by P. Duhem, it is impossible to identify a well-defined starting point for these ideas [Duhem]. Even though, in this somewhat nebulous beginning, we can identify instances in which the two structures of education join efforts and new knowledge is created. On the other hand we can distinguish the preservation of the two structures in the roots of a society of classes, as it was so clearly put by Mattoso Camara in his studies of the history of linguistics [Camara]. We will avoid being carried away by the extremely attractive and needed study of the birth of scientific ideas. We refer to [Bernal] and to [Menninger], as well as to the collection of essays of Blacker and Michael Loews [Blacker] .

The beginnings of mathematics are, in the same vein, part of a context.

But it is probably an easier task to illustrate through mathematics this link between the two educational structures we mentioned above. The very beginnings of both number words and number symbols, using the terminology of Menninger, show strong practical components as well as linguistics features which are indeed a reflection of the contemplative aspect, like for example the one-two-many, singular-dual-plural in Greek grammar, placing the step to Three as "a step across the treshold of darkness". In the same vein, the work of Claudia Zaslavsky illustrates a definitive pattern of practical considerations, in African cultures, which evolved to other forms of considerations [Zaslavsky] . As R.L. Wilder puts it, the most likely cultural component we will find among all intelligent and culture-building beings is the existence of counting processes [Wilder], and this may be an argument to displace counting as a major component of mathematical education in the context of a contemplative or speculative scheme. It would be worth to studying comparatively the development of mathematics in civilizations where the abacus was commonly used. Although taken for granted as part of theoretical mathematics in western culture, and somewhat misplaced in medieval schools, ordinary arithmetics (by this I mean the skill in performing the operations) is nothing more than a mechanical device and as such of less importance than it has been given, since medieval times, as the core of mathematical studies. In this respect, the argument of Rikitaro Fujisawa is quite interesting in discussing the teaching of arithmetic in elementary schools in Japan, published in 1912 in the Summary Report on the Teaching of Mathematics in Japan, and appearing as an Appendix to the important treatise of Yoshio Mikami on mathematics in China and Japan [Mikami]. In fact, while much less emphasis was laid upon calculation by Greek mathematicians, this was not the case with Roman civilization. The enormous emphasis given to number theory by the Pythagoreans and others, which was carried to medieval schools, has to be distinguished from ability to calculate, as Plato put it in Book VII of the Republic: "For in dealing with encampments and the occupation of strong places ... a slight modicum of geometry and calculation would suffice. What we have to consider is whether the greater and more advanced part of it tends to facilitate the apprehension of the idea of good". [Plato] . Also in Aristotle, who attributed to numbers a very physical meaning for reasons clearly discussed in [Kline Thought],

we notice a minor, almost non-mathematical role, for computation [Aristotle] . And no place for it in school.

In fact, the Greek transmitted to us two unequally developed branches of mathematics. A deductive and systematic geometry, with considerable insights into number theory, and an undeveloped arithmetic, heuristic and empirical and based essentially on calculating devices not considered as part of mathematics. This branch was pursued by the Romans, who made practical use of measuring and counting, developing several forms of abaci and finger counting. This technique, which according to Menninger is a true calculating machine, "seems to have been passed on mostly by word of mouth. No textbook describing it has been preserved from Roman times. This very fact may be the evidence of its widespread use among common or illiterate people, ... which do not require special teachers or schools" (emphasis is ours) [Menninger]. The following quote of Fibonacci, taken from Menninger, shows the dichotomy between mathematics and calculation: "If these numerals of the Indians and their place-value notation are to be thoroughly mastered ... it behooves ... to learn to count on the fingers, ...". It is quite interesting to observe the theoretical tone of the same Fibonacci in the introduction of his Liber Abaci of 1202 : "The nine numerals of the Indians are these: 9 8 7 6 5 4 3 2 1. With them and with this sign 0, any desired number can be written", which do not differ essentially from the initial paragraphs of another popular book on computing, written by Sacrobosco, and considered the most popular treatise describing operations with Arabic numerals up to the sixtheenth century. Socio-economical factors were indeed decisive in the affirmation of Indian numerals and computing with them, as a major component of post-medieval mathematics in school. See the comprehensive remarks on this in the chapter "Our Own Numerals" in Menninger's book. In fact, what was the domain of artisanal or professional education entered and dominated education in a moment when opportunity for education began to spread, universities were founded and technology was laying the grounds for the Industrial Revolution, mainly because of the introduction of the experimental method and consequently of a quantitative approach to science as the new method of natural sciences. The new social forces of the Renaissance and Reformation combined to make possible a major change in ideas in science, according to Bernal "far greater than

that in politics and religion", and replaced the whole structure of science inherited from the Greek by a new quantitative, atomic, infinite and secular view of the world. For a clear account of this period we refer to Part 4 of Bernal.

With mathematics there was a certain vacuum, a lack of new ideas. When Kline compares mathematical unproductiveness in Roman and medieval civilization, attributing their reason for extremely earthbound and heavenbound conceptions of the world, we would be more tolerant in regarding the claims of unproductiveness, and putting more emphasis on an analysis of what kind of motivation there was behind mathematics. Probably, medieval and Roman mathematicians, and the same goes for the Greeks, were closer to their future than usually thought [D'Ambrosio Campinas]. We found quite interesting the approach to historical method of Wilbur Knorr [Knorr] . We find instances of this even in modern history (see [D'Ambrosio on Kummer] for an example relating to class field theory; and on the same subject [Backmakova]). In the same vein the comments on the concept of rigor in the History of Mathematics by Kenneth R. Manning [Manning] are quite interesting. Some other cases where this lack of motivation may cause a decline in some branches of mathematics and a lack of productiveness are illustrated in the talk recently given by R.L. Wilder in the Annual Meeting of the American Mathematical Society [Wilder San Antonio] and in the paper of Charles Fisher [Fisher] on the theory of invariants. The present status of Algebraic Topology shows a similar phenomenon, as it was pointed out to me by H.B. Griffiths. We interpret this periodic decline in productiveness and lack of new ideas, in some branches of mathematics and in mathematics as a whole, as well as the opposite phenomenon, as the result of extra mathematical causes rather than as a stage of mathematics itself as would put Oswald Spengler [Spengler] , or Rubens Lintz, in their cultural approaches to mathematics [Lintz]. This way of placing mathematical thought in a global framework, looking into it as a tool which developed in man motivated by external phenomena, a finer tool than usual language to simulate those phenomena, coincides with Rene Thom's viewpoint [Thom Dialectica] . In this paper, situating the problem of the relation between mathematics and reality within the general problem of the theory of knowledge, Thom tries to place the role and function of mathematical thinking in the biological context which led from animal to man

through the development of language.

We will not resume the line of discussion which led us into some considerations about medieval mathematics, placing ourselves in another branch which developed into skill and to a large extent left untouched the mathematical edifice. Again we refer to [Zaslavsky] and the bibliography threin for illustration of the place of calculation in African cultures, and to a few remarks in [Bernal] and [Rey Pastor-Babini] for pre-columbian cultures.

Let us return to the considerations of the birth of modern science, which gained momentum with the development of a new economical order towards the end of the Middle Ages. The growing importance of towns, trades and industry forced a deviation from the economy of feudalism. The bourgeoisie, which appeared in previous centuries, imposed a form of payment which replaced, by the fifteenth century, forced service as a form of payment, and which imposed itself finally in the middle of the nineteenth century in the whole of Europe. At the same time this period laid the basis for the Industrial Revolution, heavily based on the springing up of the new science, experimental and quantified. And at the same time another component was brought into play, the opening of the frontiers of the world through navigation and conquest. This in turn facilitated the expansion of technology, by introducing new and cheap raw materials into an economy gearing increasingly into the processing of those materials.

We will not discuss the well known facts on mathematical development of this period, referring to the abundant sources available. In particular, we refer the reader to the original sources, mostly available. A good collection of important works reflecting the evolution of mathematics from Middle Ages through Nineteenth Century is found in the companion volumes edited by D.J. Struik and G. Birkhoff in the Series "Source Books in the History of Sciences" [Source Books] . We mention also the concise histories of D.J. Struik [Struik] and [Rey Pastor-Babini] as quite readable.

In this period, middle age through nineteenth century, schools went through some radical changes. While we see in the middle ages the appearance of universities, we see the development of what we have called above artisanal or professional education, resulting from a form of professional guild, in a sort of parallel structure. At the end of the middle ages, the discoveries of

new lands, the invention of the press and a new socio-economic structure, strongly influenced by the discoveries, strenghtened the position of the universities and enhanced public elementary and secondary education.

In what refers to mathematics, its place in education of this period is quite weak. When Richard Mulcaster published his "Positions", in 1581, he advanced that education should take into account "natural" development of children, and teaching should be restricted to reading, writing, drawing and music, much in the Aristotelian position. While artisanal schooling was aware of the progress underlying the scientific and technological revolution under way, elementary schools preserved their traditional look, based on Greek knowledge. Modern pedagogy owes much to the ideas advanced by Comenius in his "Magna Didactica" of 1633, which set the basis for a new and more practical schooling. At the same time, schools for the nobility acquired also a tendency for professional training. The accounts of early education in America, collected in the 32nd Yearbook of the National Council of Teachers of Mathematics of the United States give a good picture of what was going on in education in Europe and in particular of mathematics in the context of the new educational order. In fact, a most popular book was Dilworth's Schoolmaster Assistant, published as early as mid sixteenth century in England. Arithmetic there appeared as the art of computing by numbers (emphasis is ours). While in Europe a general education did not give much im - portance to mathematics and was reluctant to adopt the concept of a practical education for all, this idea prevailed in America. And mathematics was incorporated as a practical tool, reflecting what P.S. Jones and A.F. Coxford Jr. call " one of the strongest continuing forces in American education: the belief that education is necessary for the welfare of society". Nonetheless, mathematics was given less importance, irregularly taught, and often given by a private tutor. But its character is quite different from the scholastic mathematics of Greek tradition. We refer the reader to the cited yearbook for a richness of information on the early steps for the establishment of mathematics as an independent subject in America, which very much reflect the then most advanced trends in Europe. Undoubtedly, a picture of trends in European education in the eighteenth and nineteenth centuries can be very well taken from the emergence of education in the United States. We

refer also to the collection of texts assembled by James K. Bidwell and Robert G. Clason for the National Council of Teachers of Mathematics of the United States [Readings], to the comprehensive account given in Chapter 2 of Griffiths-Howson and references there contained, and to the always modern Elementary Mathematics from an Advanced Stadpoint by Klein [Klein], in particular to the Introduction and Chapter 1 of Part One. In fact, Klein's book is full of remarks which are worth meditation. Chapter 26 of Kline's book gives a good account of Mathematics and schooling in seventeenth century [Kline Thought]. We refer also to the centenary issue of the Mathematical Gazette, of March 1971, as a good source of many interesting material.

We thus reach the end of the nineteenth century, and enter the twentieth century which showed strong motivation for mathematical research, deriving from many sources, and a somewhat tranquil history ahead, mainly when we look at it after having shared it. We have never had in our profession such an undisputed position in teaching and research as we had in the first half of the century. Although there have been "schools" and somewhat opposed approaches to some problems, much of mathematics in the first half of the century followed the same ideal of placing all of it in a logic - deductive framework. The ideal setting into which mathematics was universally built into, finds in Bourbaki a good representative. In fact, much of what dominated mathematical research in the period can be traced back to last century, and it is not surprising that Hilbert's 25 problems represented a focal point in 20th century mathematics. Reading Bourbaki's history is quite illustrative [Bourbaki]. I want to bring the attention also the many historical remarks in L.C. Young's book on Calculus of Variation [Young]. Although sympatizing with the optimism shown by Hans Freudenthal when he says that "Nothing is less true [that modern mathematics is only a formal version of the old]. Not only have many old problems been solved in this century, but mathematics has also been enriched by brand-new disciplines", [Freudenthal], I can not see significant new ideas in mathematics, while agreeing with the richness of new fields of applications and sharing the optimism for a much faster and profound change resulting from the use of computers by mathematicians, which may happen with a sound educational reform.

In the same vein, I do not see significant changes in schooling. There is a major change, which is the universal acceptance of mass education, as we

have emphasized in Chapter 1. But the approach to education is practically the same, based on the ideal of doing better that our forefathers did. By adopting all the modern theories of learning, by changing curricumum structure, by devising new methodology of teaching, we are always focusing education on the hope that children "learn", that children "behave" in a certain way, that children "act according" to a pattern. In fact, modern education is claiming its competence to perform what we have called, in the beginning of the Chapter, vital education, societal education and artisanal education, and perform these functions together and very poorly. There is a striking lack of contemplative, hence creative work, although "contemplation", in a rather discouraging sense, purely repetitive and absolutely unrelated to reality, permeates schooling. It is quite illustrating to read the account on Darwin's education given by H.E. Gruber in his psychological study of scientific creativity [Gruber] .

Paradoxically, while there is an excess of conservation in both mathematics and education in the first half of the century, there is a profound richness of new directions which science and society are taking. No doubt, we are living through a second scientific revolution, with new fields of research being opened, with new tools for the exploration of nature, both in smaller and larger dimensions than man's imagination can follow. Fields as yet considered by many of the realms of religion are beginning to be established as scientific disciplines, as for example parapsychology. Techniques as yet drawing millions of readers as science fiction are being utilized in laboratories, as for example creation of life. In a more routine vein, organ transplants are today almost a triviality, and the huge computers of the 40's have been reduced in size to a cigarette pack, and the "death rays" of last century fiction writers, dramatized more recently by H.G. Wells in his "The War of the Worlds", are now a reality , fortunately being used for instant internal surgery.

In this same way, society has witnessed profound changes in the first half century. The events of the period, some still belonging to our lived experience, show a transition from one kind of society to another, from one economic order to another, and an ever increasing interaction of science and society. The balance of power that resulted in the challenge of the capitalistic system, entrenched for 300 years as the economic model for a world under European

dominance, by the socialist system, which has arisen out of the internal conflicts of capitalism itself, facilitated the recognition that the majority of the world population lives under conditions which were barely imaginable at the turn of the century. From the end of last century, foreign and colonial policies of the big powers have been largely dictated by the urge to secure great shares of the world markets for the products of the growing national industrial trusts. Largely favoured by the balance of power which grew out of two World Wars, newly independnet nations entered the picture of world economic equilibrium, increasingly playing a more consequential role. Firstly having achieved political independence, these nations struggle for economic and cultural independence and search for their somewhat forgotten moral and cultural identity. It is unnecessary to recall the changes in the world economic pattern resulting from this new political picture. And the same goes for the attitudes and values of society as a whole, with enormous reflection in schooling.

We close this chapter within the framework of profound social, political, economical, behavioural changes we are living through, intermingled with not less profound changes in science and technology, and the resulting need of questioning the moral values which necessarily play a role in this intermingling.

Mathematics and mathematical education and part of this complex.

3. THE ROLE AND NATURE OF MATHEMATICS.

This chapter could very well carry the title "What is mathematics?". I borrow the idea of beginning with dictionaries from Professor L. Santaló, using his example from the Dictionary of the Royal Spanish Academy : "Mathematics is the science that deals with quantity", while "Quantity is all that is passive of enlarging and reducing and can, as a consequence, be measured and enumerated" and "Science is the exact knowledge of things by its principles and causes". Or from Webster : "Mathematics is the science dealing with quantity, form, measurement, and arrangement and in particular with the methods for discovering by concepts and symbols, the properties and interrelationships of quantities and magnitudes". While "Quantity is anything which can be multiplied, divided or measured; anything to which mathematical processes are applicable". Or from Isidore de Seville's

Etymologies, written circa 600: "Mathematics ... considers abstracts quantity. For that is abstract quantity which we treat by reason alone, separating it by the intellect from the material or from other nonessentials, as for example, equal, unequal, or the like". Of course, we will not proceed in this exercise, but I certainly would like to have access to a collection of definitions of mathematics through the times and try to identify in those definitions what mathematics has been thought of and what are the expectations of society with respect to mathematics, how ample is its reach outside the inner circle of its practicioners, and how meaningful has been its teaching. Because we can hardly see mathematics outside of a context, we fully agree with A.G. Howson when he says that "education, particularly at primary and secondary levels, cannot ever be "culture-free" : it is rooted in a particular society. "Culture-free" mathematics would be a largely useless appendix".

Mathematics and some non-mathematicians can identify with ease mathematics in intellectual experiences, and can distinguish mathematical reasoning and method in practically any situation where they are either present or potentially inserted. I cannot but agree with the position of René Thom in [Thom Dialectica] in placing mathematical thinking as a finer language to simulate external phenomena than usual languages. This places mathematics within the ample context of scientific reasoning and method, demanding for an analysis ofits role and nature to place it in a general context and in its relation with other languages which try to simulate a reality. Indeed, the process of mathematical creation proceeds like any form of creation, as the result of some internal force which we can not identify, while the object of mathematics is not the result of a purely sensorial experience, a pure observation, but the combination of this with some process of abstraction and correlation of ideas. This is what René Thom calls in [Thom Critique] "something like an alternation of reducionist and structuralist tendencies". Having assumed that mathematics is itself a morphologic discipline, since it studies forms and structures which appear as given, the reductionists and structuralists tendencies are basically related to what we have called in chapter 2 motivation, and to what we distinguish as consolidation or systematization of knowledge. We refer to [Thom Critique]

for a detailed explanation of his ideas.

Other attempts to explain how mathematics progresses seem to agree on a peculiar state of mind which leads to the creation of new mathematics. To describe this peculiar way of creativity is very difficult. There are attempts at a systematic description of it. In a recent paper, Elaine Koppelman [Koppelman] detailed the major steps which she claims, together with small advances, make mathematics progress. She distinguishes six major categories of steps which lead to new mathematics, and names them: ordering, transplantation, fission, fusion, conceptual generalization and pure invention. She illustrates the first five with examples taken from the history of mathematics and ends with the assertion that "pure invention has probably never occurred". The lively discussion that follows her paper shows the incredulity of some as to the possibility of a taxonomy to describe mathematical creation. In the words of Garrett Birkhoff : "The idea of developing a new science of historical analysis which is a pure science all by itself is a dangerous trend ... It is useful to have a rough descriptive classification of the different ways in which new theories are born, but we should not exagerate the importance of such taxonomies". While not inclined towards taxonomies, and in particular to the one presented by E. Koppelman, I see in the historical analysis of mathematical creation, approached in an interdisciplinary context, a most fundamental component for mathematical education. Certainly, the remarks of G. Birkhoff should not be interpreted as discouraging attempts for an historical analysis.

An important source for the analysis of scientific creativity is provided by scientists themselves. In fact, we have a few autobiographical essays written by mathematicians, and I would urge more of these essays to be written. Not only this, but more studies on mathematicians are needed. Fortunately, we notice a growing number of books and articles in this line.

Now, if we place ourselves in the position of a glance to the future with respect to mathematics, it is very difficult to identify some trends. Of course, we have to place, as a major force in shaping the future of mathematics, its growing relationship with other scientific fields. I found quite interesting a Symposium held during the 138th Meeting of the American Association for the

Advancement of Sciences (Philadelphia, 1971), where Harold Grad states that all mathematics has been or can be applied constructively to some natural phenomenon, and that probably the major road block to actual applications of inherently applicable mathematics is the incredible obscure way in which mathematics is conventionally presented. He also claims that a person's interest will determine the direction that he will go. By the way, the same H. Grad advances some hints on future directions for mathematical research in [Grad]. We entirely agree that this is the case - in fact another way of putting what we have called motivation in Chapter 2; and consequently place much of the considerations about the future of mathematics in its relations with other disciplines and a marked trend towards interdisciplinarity. It is to be considered that countries like for example the People's Republic of China, with close to a million individuals, a strong mathematical tradition and a growing economy, are directing mathematical research towards a marked interdisciplinary framework. An analysis of priority areas for research in the United States shows a growing trend towards inter - disciplinary research. As it was remarked to me, in private correspondence, by Peter Hilton, reacting to my comments that there is a need in developing countries to "qualify" mathematics research, the same way as is done in the United States and in Europe through funding agencies and hiring policies, "it is the responsibility of mathematicians to direct the attention of their students to the important areas of applications of mathematics; and also to make that their students understand the methodology of applying mathematics". In fact, what I see as the major factors determining the future of mathematics research are those factors which we may call "external", but which will certainly shape priorities in mathematical research. Very significant are the references made by social scientists to catastrophe theory, with the accepted view that this will provide an effective tool for the study of their discipline. The "defensive" tone of some mathematicians, as examplified by the final words of Carlo Pucci in the closing of the "Convegno su Rapporti tra ricerca matematica pura e ricerca matematica applicata" held in Siena, 1973; "We talked mainly about applied research; this does not mean a devaluation of the role of autonomous research;... which has a cultural and critical function which interests everybody as a natural

and indispensable sustaining of the freedom for all", reflects an awareness of the strong influence of applications in mathematical research. Certainly, remarks like this would have found no place sometime ago, when what was meant by research it was taken for granted. This is challenged by the high costs of supporting the scientific establishment, plus some internal crises, as put by Errett Bishop [Bishop]. Although hardly meriting being identified as a trend, Bishop's remarks bring attention to a certain isolation within the scientific community which mathematics and mathematicians have enjoyed, and some try to preserve. There is no value judgement in my remark, only the recognition of a fact, and of a trend towards a new relationship between mathematicians and non - mathematicians. This is obviously not bad at all, and it may be quite stimulating as it was expressed by René Thom in his Critique paper: "the contacts which I have been keeping with specialists of disciplines in the most diverse areas have shown me how little the scientist, in general, is preoccupied with the goals of his own discipline, consequently of Science ...". It is not a matter of choice, this changing attitude. As a consequence, new foci of interest for research mathematicians will come through an osmotic process. This process goes both ways and the appearance of completely new scientific fields, bearing mixed characteristics of mathematics as we know it today and of other disciplines, is quite possible. The role of computers in those new scientific disciplines will probably be a major one.

Is this a future that is within the reach of our life time? Probably so, but certainly it is in the life span of children in our schools. The implications of the above considerations for schooling are of considerable magnitude and may strongly affect mathematical education. Certainly the emphasis should be changed from what is taught, that is, from curricula, syllabi and contents, to methodology and to a new classroom environment, both geared towards creativity in its most ample sense. Something must be said on the process of discovery and creativity. In fact, this entire change was magnificently captured by Professor H. Freudenthal when he replaced the famous principle of Comeniu's didactics : "The best way to teach an activity is to show it" by his own principle "The best way to learn an activity is to perform it". Indeed, the focus in schooling is on

learner. In particular, mathematical creativity is gained throughout school years if mathematics is done in those years. Creativiness springs out as the core of mathematical education, but has a much broader reach. It is by no means restricted to mathematics. It is a global attitude, hopefully enhanced by mathematics. This is very well put by J.A. Dunn in his review paper [Dunn] , which contains an excellent bibliography on mathematical creativity. As Dunn says, in mathematical activity in the classroom the subject-matter is known, while mathematical activity in professional life is like a creative art. While recognising the importance of learning theory for the considerations relevant to this chapter, I will not touch upon the subject, referring to section B-4 of this congress and to the excellent bibliography assembled in [Dunn] and [Dunn 2] . In this second paper, an interesting analysis of criteria of creativity is given.

This psychological setting needs an appropriate school structure to operate, as well as instruments for generating motivation, which will be the subjects of the following chapters.

Before finishing this chapter, let me add that I could hardly claim the observations made above represent more than my personal view of the subject, supported by isolated remarks which add up, in my understanding, to the views expressed in the chapter. My perception of the direction mathematical research is going in fits ideally in what I see as a suitable scientific and technological structure for developing countries, quite dynamical and auto-modifying to serve constantly changing needs of society. This scientific and technological structure goes side by side with an educational structure which must have the same dynamism and power to reform itself during the process, to fit the expectations of the society in which it is placed as an general part of it. These expectations will be examined in the next chapter.

4. WHAT DOES SOCIETY EXPECT FROM MATHEMATICAL EDUCATION?

We reach the core of our paper in what regards objectives of mathematical education, and will look into the question of "Why teach mathematics?" in the framework built in the previous chapters. Bent Christiansen in his outline of

major development in mathematical education in the period 1960-76 [Christiansen] mentions as categories of the present goals of mathematical education in the individual countries those related to the nature and role of mathematics, to the learner's working methods and to his use of mathematical methods and principles, and to knowledge and skills. In these broad categories, many of the specific goals can be inserted. But we would rather tend not to categorize goals and look into the subject in a global way, emphasizing that "our primary concern should be the improvement of the overall education of the pupil - that the teacher's job is to educate and not merely to instruct", in the words of Griffith and Howson. Of course, there is no contradiction between this approach and the categorized way of looking into goals. But I feel we can get more flexibility and dynamism in education, mainly in mathematical education, if we look into the matter in a global setting. Again, there is no contradiction with a more stratified approach, like for example the one presented by W. Servais [Servais], in which we find a listing of 16 items categorized as general objectives for building up an intellectual, esthetic and moral background. Among those we mention as examples, "to acquire a constructive critical sense" and "to activate a taste in the expressions of thought: clearness, order, conciseness, elegance". We would rather place those as consequence, as subproducts, as natural attitudes within a context, and not as objectives, which a fortiori imply more immediate aims. Certainly, mathematical education is not the only route to achieve this kind of behavior which, in an order of values upon which western society traditionally rests, may be regarded as an ideal of excellence. And probably, it may not be the best route available. Indeed, we notice some changing attitude towards a somewhat undisputed place of mathematics in the educational context. As we read in the NACOME Report [NACOME], an "Overview and Analysis of School Mathematics in the United States", published in 1975 by the National Advisory Committee on Mathematical Education of the Conference Board of the Mathematical Sciences, "current cultural preoccupations no long award mathematical, scientific and technological disciplines the first place of honor (and funding) that has been the case for many years". I do not see much point in trying to defend the place of mathematics, as in the same report which says that "whatever alternative fields of scholarship take precedence over mathematics ..., our world remains a scientific, technological, and industrial

world and mathematics remains essential to its caretaking and improvement". This, in itself represents an objective for mathematical education which is, obviously, open to a deeper and more menacing criticism. As is recognized in the same report, education has been conditioned by recent cultural changes, which deeply challenge the scientific, technological and industrial world, on which are based many of the objectives for mathematical education, as put in the NACOME Report. I see this process of making our societies, what their caracteristics are, more and more based on advanced science and technology, an irreversible one and indeed a favourable one. No point in rejecting scientific and technological progress on the basis of some bad uses that have been made of them.

Hence, as a premise to the continuation of the discussion of this chapter, it must be made clear what kind of societies we are talking about when trying to locate mathematical education into these societies. We must accept Utopia as our ideal society, which I feel quite legitimate for us to think about when discussing the future, and education leads to this future. It is irrelevant to discuss details of Utopia. No one will disagree that this Utopia, in many of its several forms, includes justice, freedom, high quality of life, et. Towards Utopia humanity has been building up the various kinds of societies, and has been accumulating knowledge, technology and destroying myths. In this context, education has been evolving. Mathematical education is placed in this context.

Different roles are expected for mathematical education in the various societies which are today confronting and influencing each other. In each of these societies, technology plays a fundamental role. Most appropriate was the scope of a projet conducted at the University of Bielefeld under the title "Mathematics in industrial society". The works of the project went under way under the hypothesis that in general in the Federal Republic of Germany the following occurs: 1. a mathematical student is to be prepared for a profession and approximately one third of all math-graduates will be working in some branch of industry; 2. when choosing mathematics as a major field of studies, the student does not consider aspects of possible professional practice; 3. during his studies, the student is not critically confronted with his professional

situation to come to develop a sense of "social consciousness" ; 4. the mathematical fields being related to practical application are not integrated in the course of studies in a convenient way.

This project reflects, in its conception, an eminently utilitarian view of mathematical education, not basically different from the view underlying the observations of the NACOME Report. In a round table held during the International Congress of Mathematicians in Vancouver, 1974, originated by the Bielefeld project, Professor B. Gnedenko, of the USSR, very clearly emphasized the trend in universities of the Soviet Union towards what he called a "blocking of mathematics ... and instruction only in the pure direction without applications", which, according to his sayings and plans, did not correspond to what industrialized Soviet society expected of mathematical university education. We would be able to multiply examples and evidence showing a growing need of trained mathematicians for the process of development and the use of technology in its various stages. The examples above can be found in most countries, showing concern of mathematicians about the distance found between what is taught or researched in mathematics and what is applied. This growing concern and growing emphasis on applications is well expressed by J.J. Seidel in referring to the mathematical engineering curricula in The Netherlands: "Applied mathematics was [Before 1950] mainly developed by representatives from other disciplines, such as physics and engineering. However, in the last 15 years there has been a considerable change". [Seidel]. Either accepting Peter Hilton's defensive posture that "Applied Mathematics is not an alternative to pure mathematics. It is the superimposition of an extra and very difficult skill on top of the already difficult problem of learning mathematics", or his much milder "doing applied mathematics is doing mathematics within a context of concern for the solution of scientific, non-mathematical problems", [Hilton] or Morris Kline's agressive position in saying that "Mathematics is a simple subject compared to economics, psychology or physics. It is a narrow subject ... Moreover, mathematics <u>per se</u> does not deal with human beings and the complex problems that dealing with human beings pose". [Kline Johnny], we have to recognize that society at large expects much more than the view that Peter Hilton claims, in the same paper, as " the

strong and probably valid impression among university administrators and many mathematicians themselves that ... is mathematics itself, for itself, to be regarded as relevant".

In fact, we have digressed and brought the subjects to the long and sterile dispute about pure versus applied mathematics or its variant, mathematics versus applications. As Chandler Davis pointed out in the round table discussion held during the Vancouver Congress, it is wrong to assume that the improvement of relations between the "Mathematical and the outside non - mathematical world" depends on changing the ratio of pure and applied mathematics or of mathematics and its applications in the schools. It is mainly a matter of attitude, which has to be generated in early schooling.

At this point I claim that society on the whole expects mathematicians to be competent professionals, capable of justifying why they are being paid to do mathematics. This is a very crude, unsophisticated way of putting it. Research mathematicians as a class have enjoyed an unprecedented prestigious position after the 50's, with the expectation that much, if not all, of what was being done on mathematical research would, in an almost immediate future, be translated into technological progress and development. Practically no account of what was going on in mathematical research was ever asked for. The few instances of challenge have originated in the United States in the 60's during the dispute over Smale's grant, in the period after May 1968 in Europe and also in the United States, and in particular in the People's Republic of China, during the period called "Cultural Revolution" (see [Swetz China]) . And now the dispute over peer reviewing going on in the United States reflects much of the end of this "credibility" period.

Mathematical education is a reflection of the position mathematics and mathematicians enjoy in society. While doing speculative or contempative education directed to an elite, leaving professional training to a parallel structure, the expectation of society regarding education was much more directed to individual fulfillment. With mass education and the combining of professional training into the educational system, society expects much more. We notice in most of the listing of objectives a strong emphasis on societal behaviour and reaction to the external world. For example, the document prepared by the

"Association des Professeurs de l'Enseignement Public", of France, under the responsibility of Michel de Cointet and Michèle Couchan lists with great emphasis, the development of attitudes as a major objective of mathematical education. The student should be able to, as an objective of his schooling : 1. analyse different components of a situation; 2. recognize analogous situations; 3. choose a strategy adapted to a situation; 4. make himself understood by others; 5. have a critical attitude; 6. construct simple deductions and construct a chain of deductions; 7. predict a result and generalize; 8. construct, eventually, a simple model; 9. participate in a collective task.

Very similar are the objectives listed in the curriculum guide prepared for the Secretary of Education of the State of Sao Paulo, in Brazil: 1. develop the capability of: analysing, relating, comparing, classifying, ordering, synthesizing, evaluating, abstracting, generalizing, creating; 2. develop the habit of studying, of rigor and precision, of order and clearness, of correct use of language, of conciseness, of endurance in obtaining solutions for problems attacked and for criticism and discussion of results obtained; 3. acquire especific abilities for: measuring and comparing measures, calculating, constructing and consulting tables, tracing and interpreting graphs, utilizing and interpreting correctly symbology and terminology of mathematics; 4. acquire information and knowledge about several concepts and methods utilized in mathematics; 5. develop the capability of obtaining, from given conditions,valid results in new situations, utilizing deductive method; 6. recognize the inter - relation among the several fields of mathematics.

With minor variation, this is the pattern of listed objectives we have found all over the world. Whatever the set of objectives, questions like "Why?", "To whom?", "How?" are present in any effort to build a better curriculum. In the context of developing countries, the major efforts to bring about a more modern curriculum and alternative solutions based on a local answer to the classical three questions are, in many instances, less successful than foreign ideas and models brought into the country without the appropriate criticism and adequacy. The "new math" movement in Brazil is a good example of this. While a deep study of curriculum was urged back in 1957, calling for taking into account the impact of the then recent studies of Piaget, (see [D'Ambrosio Ensino]),

foreign "modelled" modern mathematics appeared in an instantaneous way in 1961 and almost immediately dominated the scene. The situation is not different in other developing countries. In other cases, not only ideas are brought into the country, but materials are "adapted" or "specially designed" to fit the goals and objectives of their mathematical education. An illustrative example is the Entebbe Project. On the other hand, we should mention some corageous attitudes like the efforts carried out by the Republic of Mali to set up their own educational structure. The resulting mathematics curriculum for primary school and the enormous implementation scheme set up under the leadership of Bakary Traore is an example of endurance, in the face of quite adverse conditions. No doubt, examples of both kinds can be easily multiplied.

We would rather proceed in commenting on some of the more global objectives which reflect a philosophical attitude in response to the role mathematics is expected to play in the context of general education. Again, we see the two basic aspects of education, as a professional or utilitarian discipline and as part of a general cultural framework, aiming at the understanding and mastering of the world, in a speculative way. I will elaborate on the first aspect, which is in a more utilitarian vein and probably much more affected by the introduction of calculating machines and computers, and which goes much in the direction of a training. This has probably to be associated with professional schools and the main objective is to develop skills. Again, the aim in this case is a direct approach to applications, to reality, in a most unsophisticated way. I see no point in insisting on theorectical considerations, no concern at all in providing bases for the tools which are given to the students. The aim is to provide skills in calculating, probably done in a much better way with hand-held calculators, and according to specific needs, with more advanced techniques. I recently had the opportunity of engaging in a conversation with a curriculum specialist of the Ministry of Education of Ethiopia, on elementary school mathematics. I was reassured of the priority of this skills oriented mathematical training as a parallel track for an alternative, creative, approach. I see the learning of this practical mathematics in the same way as the learning of a language, allowing for everyday usefulness. Again, this aims at a professional training and should be placed in the context discussed in

Chapter 2. This kind of instruction, to which Skinner's techniques can be successfully applied, is common in military schools, and reached a high degree of efficiency in war times. The proliferation of this kind of schooling that accompanied the beginning of the industrial era, which caused Nietzsche's bitter reaction in his 1872 conferences in Basel [Nietzsche] , was absorbed into mass education and, in the case of mathematics, became the major component of mathematical education. In fact, these objectives were clearly stated in the 1899 Report of the National Education Association of the United States: "... the main emphasis should be given to such topics as are useful in later work ... there should be applications of algebra, geometry, and arithmetic to each other, and to various sciences and the practical affairs of life". [History NCTM] . And in current times, there is an identified trend in "an increased emphasis on usefulness (in an ordinary daily life sense) and on applicability of the outcomes of mathematics teaching". [Christiansen Europe] . In fact, a special effort was devoted by UNESCO to bring more immediatelt applicable mathematics to elementary and secondary schools in Latin America [UNESCO Montevideo]. No doubt this is an identified, and quite respectable, goal for mathematical education , but unfortunately not accepted in its full reach and purity by mathematical educators. In fact, while Leibniz strived "for a mechanization of purely technical aspects, in particular the process of computation, hence his perennial struggle for a calculating machine equally useful for all four species of computation" [Hofmann], we still find some reluctance in the full adoption of hand calculators and computers in mathematical education.

While this practical objective of mathematical education anchored itself in mass education, it was confounded with other objectives, and in fact assumed the place of mathematical education par excellence. The so-called "modern mathematics" was nothing but a reaction to this one-sidedness of mathematical education. Unfortunately, new math came to replace practical mathematics, and what we see today is a struggle to fit one approach to mathematical education into two absolutely different objectives.

The second kind of mathematical education, which was displaced after the industrial revolution and could not find its way back to the general context of science education, is part of the effort for developing and inquisitive,

creative, free individual, fulfilling the aim of education as "The acquisition of the art of the utilization of knowledge" [Whitehead Aims]. It is necessarily related to global reality. Again, in the words of Whitehead, "The solution which I am urging is to eradicate the fatal disconnection of subjects which kills the vitality of our modern curriculum. There is only one subject-matter for education, and that is life in all its manifestations. Instead of this single unity, we offer children - Algebra, from which nothing follows; Geometry, from which nothing follows; Science, from which nothing follows; ... The best that can be said of it is, that it is a rapid table of contents which a deity migth run over in his mind while he was thinking of creating a world". The objective of this much more ustructured creative form of mathematical education, closer to the real problems, with lots of plausible reasoning in the line of G. Polya (see [Polya]) and the child's need to live and grow in his own way, following the approach of H. Whitney (see [Whitney]), is to place mathematics again in its position of an extremely convenient and successful language to simulate the real world. The objective is to produce new mathematics, to create new theories, to help in the solution of the new problems which are only now being identified and recognized. The objective is to give mathematics a new dimension that will fit a new world and societies which are closer to each other and looking for their Utopia. For this we need new mathematics, we need all the creative power of youth, we need new forms of thought which we can not envisage. The primary objective of mathematical education is not to perpetuate knowledge or to push a little further existing knowledge. This will go on or will fade away as a result. But we have to foster the creation of new knowledge. Teaching is not the essential goal of this creative or contemplative form of mathematical education. It is fundamentally to propiciate a favourable setting for the creation of new knowledge. Strategies for this will be discussed in the next chapter. Let us repeat that this objective runs parallel, and is in fact aided, by the practical or immediate goal of furnishing mathematical techniques and tools for professional or daily activities. This goal should be achieved in a most direct, practical, unsophisticated and fast way. Yet, the most advanced theories should be made available to utilization by a large number of individuals and groups

which may need them for practical purposes. It is indeed a massive effort to bring advanced mathematics to a readily applicable form.

In developing countries, this dual aspect of mathematical education deserves a much more explicit definition in the statements of their goals and objectives. While large and well established educational systems have mechanisms to identify what might be called "mathematical talent" and to provide, through their traditional cultural establishment, like music, national monuments and personalities, etc., some stimulation for the pursuit of scholarship in mathematics, developing countries need a special strategy. Developed countries show the presence of the two objectives in statements like the following by V.A. Krutetskii, head of the section of abilities of the department of educational psychology of the Institute of Psychology of the Soviet Union: "The value of mathematics as a science is very broad ... It would be hard to find any field of knowledge where mathematics did not play a role ... Mathematics is necessary in the work of engineers and technicians, and is necessary for many skilled professions ..., acquiring special significance in connection with the gigantic growth of science and technology in our country ... Our government is highly interested in the appearance and cultivation of mathematical talent. We aim to develop the mathematical ability of every student in accordance with his potentialities ... Not everyone can or should become a mathematician, but mathematics will be necessary in almost everybody's life". [Krutetskii]. This is put in a more explicit way in the NACOME Report, in stating that "Contrasting syllabi and texts for the different student audiences, one is quickly struck by the emergence of two distinct mathematical cultures - one focusing on broad structural concepts and heuristic methods, the other on computational skill and specific techniques applicable to problems or everyday life and specialized trades". Coherently with the tone of its previous remarks, where it was avoided the recognition that there are less able students and that this lack of ability is related to socio-economic and cultural background, the Report asks if the dichotomy is inevitable and appropriate. But later on it takes a positive stance, recommending that for those who have been unsuccessful in acquiring computational facility in the fundamental arithmetic operations, the use of electronic calculators will provide for their needs in arithmetical skills.

The recognition of, insisting on C.P. Snow's language, two different mathematical cultures, has been avoided in the grounds of not generating a scientific elite, as if social unbalance and class privileges reside in this fact. Mainly in developing countries, but also in many developed countries, there is an insistance in putting together two absolutely different approaches as far as their objectives are concerned with mathematical education, hence doing both poorly. For developing countries, to recognize those two different objectives and goals, and to assign appropriate priorities to each one, is probably the most important task for their educational system. No doubt, a healthy society will place adequate values on each of those functions, and they will fertilize each other to build up science and technology without losing moral and cultural values. Machines and techniques can be brought to a foreign culture without necessarily ruining it, but ideas and values can not be generated in a culture free context.

5. AGAIN MATHEMATICS IN SCHOOL.

In this chapter we will propose some strategies which allow for parallel running of the two apparently conflicting objectives for mathematical education, the one based on the concept of mathematics as a utilitarian body of skills and techniques, designed and taught to satisfy social needs, and the other looking into mathematics as a component of the large body of models of thought and languages which simulate external phenomena.

Again, we have to reason in an ideal framework, and place our aims without much concern about difficulties in attaining these aims, and then apply an approximative scheme, which points to the stated aim. I strongly believe in the tactics of placing the goals very high, sometimes in the domain of possible unreachable, and as a first step to achieve only part of it. And immediately going to a next step ... No one will deny the residual benefits of the new math movement, nor the healthy effects of the sometimes virulent attacks on it, and as a result the not-so-new look of good old mathematics!

In fact, a new educational structure is finding its place in the world. A major aspect of the change refers to numbers . We have to agree with Dragoljub

Najman when he says that "There is a fundamental disparity between institutions of higher learning which had, ... , six million students and which will have tomorrow eighty million or one hundred and ten millio. ... there is an enormous qualitative difference between one situation where, in the entire world, twenty years ago, there were only six hundred thousand professors in higher education and, ..., in a few years [later] close to four million". [Najman] . The same kind of remark would apply to other levels of education. But I believe the real impact comes through higher education. It is on this level that the prefigurative culture so well pictured by Margaret Mead [Mead] will manifest itself now confronting masses of dimensions never thought of. As M. Mead puts it, "The fact that ... older scientists send their pupils to other laboratories to work on newer problems - changes nothing ... Even in science, where we have tried to build in the expectation of discovery and innovations, students learn from old models, and normal young scientists work to fill in blank spaces in accepted paradigms. In today's accelerating rate of scientific discovery, the old are outmoded rapidly and replaced by near peers, but still within a framework of authority". There should be no doubts in our minds that the mass of students entering the institutions of higher learning, if only because of sheer probabilistic considerations, plus the necessarily lower age of faculty, resulting from their dramatic numerical increase, will unbalance the power structure. The concept of "cogobierno" which has marked much of the conflicting issues in Latin America since its declaration during the Cordoba movement, back in 1918, and which has taken many forms elsewhere, can not be interpreted as a natural consequence of those large numbers. An analysis of those facts and consequences, placed in the Latin American problematic, can be found in [Ribeiro] . We are passing from the era of higher learning to the era of university, in its full conception. Again, as a pure consequence of numbers involved, we are closer to the concept of a learning community, a global education context. This structure fosters the creation of new knowledge, which is permanently brought to a readily accessible form, readily adaptable to uses in service of the comunity, through a well set system of integrated university and production centers. Examples of this we find in some cooperative programs established in the United States, theorized in the collection of essays under the title Campus 1980 , edited by Alvin C. Eurich

[Kurich] and, in regard to graduate education, in the report Scholarship for Society, sponsored by the Council of Graduate Schools in the United States [Scholarship]. It seems that modern Chinese approach to higher education is very much in this line, from what I could infer reading some reports from recent visitors to the People's Republic of China.

Within this context, mathematics will probably find the revival of its creative power. Naturally, what is needed is a departure from structured curricula, based on contents which is decided and fixed a priori. In a much more dynamic approach, teaching is structured in a series of interdisciplinary activities, resting strongly on motivation from which units of contents, in various levels of rigour and variable duration, to a large extent self-contained, are generated. Very much in the concept of Whitehead of eradicating the fatal disconnection of subjects, Jean Piaget claims that "an increasingly large place must be reserved for new points of view - interdisciplinary by nature - like those which are being developed today by psycho-linguistics, decision theory, economic psychology and sociology, and so on. This however does not means more class hours ..., but rather ... a systematic broadening of outlook", [Piaget]. A large emphasis is to be placed in pedagogical efforts to bring advanced theories to accessible presentation to audiences, with various degrees of background and different motivation. Efforts like for example Solomon Lefschetz's lectures on the applications of algebraic topology, or C.P. Bruter's Topologie et Perception, or Ian Stewart's Galois Theory or Silvanus Thompson's Calculus made easy . As Professor Santaló puts it, "there are topics and advanced mathematics which ... should be brought to secondary school or university teaching for non-mathematicians ... This constitutes a major challenge for pedagogy and the didactics of mathematics. It should be aimed at teaching those subjects without intermediate steps, even at the expense of formal rigour being replaced by an intuitive approach". [Santaló]. But this can not be achieved within a pre-fixed curricular context. A moderate approach to this unstructured curriculum described by E. C. Zeeman in his intervention during the round table on the Bielefeld questionnaire held in Vancouver. His approach is based on two principles: first, let the students do what he likes; second, induce him to do at least one thing sufficiently deeply to affect him for life. Another principle which he brings to

issue is that "people are more important than structure, so you get people first and then you set up the structure around them and the type of cooperation will depend on the haphazardness of the individuals you have". This form of unstructured curriculum can provide for the learning environment we propose. On the other hand, some techniques of methodology of access to information become more and more needed. Information retrieval and the proliferation of audio - visual technology applied to education seems to be impossible to postpone. Risking sounding paradoxical, I claim poor countries should look into modern educational technology as playing a key role in their future. In particular, much mathematics information can be placed in cassetes, both audio and video, and be made available to a practically unlimited clientele, of different regions of the world, while problem posing and the motivation for those stored units of knowledge can not be generated other than locally. The two forms of education, which we called professional or artisanal and speculative or contemplative, follow the same pattern and reinforce each other, as part of a same context. Strong emphasis on problem solving, both leading to a more speculative development of attitude, like [Polya] and [Polya-Szegö], or to a professional attitude, like for example the set of problems assembled by Ben Noble [Noble] may provide a good starting point for this motivation oriented approach. As F.J. Almgren Jr. whose works on Plateau's problem give a good example of the combination of very intuitive ideas, in a real and even experimental context, with highly sophisticated techniques, says: "a student of mathematics should come into contact with a broad range of ideas in many different fields even tough at any particular time many of these may be understood only superficially. I believe that much of our mathematical thinking is by analogy and that we need as many things in our heads to make analogies with as possible. I know of no particu - larly good way to be very selective in this process of accumulating ideas". Certainly, the creation of an atmosphere of enquire, interdisciplinarian and not pressured by a priori standardized curricula will be much more favourable to the creative process we expect from mathematics.

I have no doubt this approach also responds more realistically and more immediately to the needs of developing countries. A more detailed description of such a model is to be found in [D'Ambrosio Rabat] . A larger scale experiment, akin to this model, is the project known as "CPS Bamako", a

UNESCO sponsored interdisciplinary graduate program in Mali, now in its sixth year. A more general experiment, in the sense of being interdisciplinary and with the more immediate aim of generating foci of reform at the primary, secondary and undergraduate levels, is under way in Latin America, under joint sponsorship of the Organization of American States and Brazilian Ministry of Education at the State University of Campinas, Brazil. With the initial goal of preparing, at a Master's degree level, 128 specialists in science and mathematical education, from all over Latin America, in a six year period, the project represents a testing ground for the motivation based approach, with traditional contents appearing as a result of the evolution of the course.

We are convinced that a sound strategy of reform is to implement it on all levels simultaneously. As Professor Howard Fehr said, in his conference delivered in the IV Interamerican Conference on Mathematical Education, December 1975, in Caracas, "the teachers who are responsible for improvement of their profession must possess an adaptivity to change ... they must be aware of new goals of teaching brought about by new technology and the need to experiment with these new intrusions into an existing program". A realistic and dynamic approach to change means, in its essence, built-in mechanisms of change, in which teachers and learners are brought into a single organic component.

In fact, I believe this motivation based approach does not add to existing learning theories. Lack of time to pursue this further did not allow me to elaborate on psycho-pedagogical aspects of the proposed approach. The exhaustive book of Erich Wittmann is certainly a guide for an analysis of the approach from the point of view of mathematics learning [Wittmann] .

We will briefly discuss the approach with respect to primary and secondary teaching. In fact, one of the main difficulties we face in the experiments described above is a reaction to change. Professor Fehr's remark carries in it an objective of prime importance. At the university level, we work with adults, conditioned to a pattern of schooling, and one of the first tasks in our experiments has been the demystification process. On more elementary levels, mainly in primary schools, this is not the case, which is in itself a favourable aspect. Indeed, there is much more going on in this direction than we would be able to mention. The "A" sections have discussed many models. In particular,

Section B-1 has brought attention to many curricular projects which have the characteristics of our motivation based approach.

As a conclusion, it must be emphasized that a strategy for mathematics teaching is an integrating part of the definition of its objectives and goals. And these objectives and goals must necessarily be regarded as part of the targets of the society in which they are inserted. In this context, mathematics itself is a strategy in the overall societal aspirations. The same as education, mathematics is inserted, together with all other forms of knowledge, in the set of strategies created and developed to reach Utopia. Not necessarily part of Utopia.

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